Fitting improved hazard models for SAC/FEMA-compatible seismic analysis

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Introduction

- Emergence of performance-based earthquake engineering (PBEE) with the development of SAC project by FEMA to evaluate performance of structures in a probabilistic manner
- Introduction of a power law to characterize seismic hazard

$$H(s) = k_o s^{-k}$$

- Adequacy of power law questioned!
- Accurate quantification of seismic hazard as one of the cornerstones of earthquake engineering with a revised expression

$$H(s) = k_o \exp\left(-k_2 \ln^2 s - k_1 \ln s\right)$$

- Some robust and objective ways to quantify the coefficients are needed!

Probabilistic Basis for 2000 SAC Federal Emergency Management Agency Steel Moment Frame Guidelines

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Abstract: This paper presents a formal probabilistic framework for seismic design and assessment of structures and its application to steel moment-resisting frame buildings. This is the probabilistic basis for the 2000 SAC Federal Emergency Management Agency (FEMA) steel moment frame guidelines. The framework is based on realizing a performance objective expressed as the probability of exceeding a specified performance level. Performance levels are quantified as expressions relating generic structural variables "demand" and "capacity" that are described by nonlinear, dynamic displacements of the structure. Common probabilistic analysis tools are used to convolve both the randomness and uncertainty characteristics of ground motion intensity, structural "demand," and structural system "capacity" in order to derive an expression for the probability of achieving the specified performance level. Stemming from this probabilistic framework, a safety-checking format of the conventional "load and resistance factor" kind is developed with load and resistance terms being replaced by the more generic terms "demand" and "capacity," respectively. This framework also allows for a format based on quantitative confidence statements regarding the likelihood of the performance objective being met. This format has been adopted in the SAC/FEMA guidelines.

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CE Database keywords: Steel frames; Probabilistic methods; Moments; Seismic hazard.

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Derivation of new SAC/FEMA performance evaluation solutions with second-order hazard approximation

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SUMMARY

A novel set of SAC/FEMA-style closed-form expressions is presented to accurately assess structural safety under seismic action. Such solutions allow the practical evaluation of the risk integral convolving seismic hazard and structural response by using a number of idealizations to achieve a simple analytical form. The most heavily criticized approximation of the SAC/FEMA formats is the first-order power-law fit of the hazard curve. It results to unacceptable errors whenever the curvature of the hazard function becomes significant. Adopting a second-order fit, instead, allows capturing the hazard curvature at the cost of necessitating new analytic forms. The new set of equations is a complete replacement of the original, enabling (a) accurate estimation of the mean annual frequency of limit-state exceedance and (b) safety checking for specified performance objectives in a code-compatible format. More importantly, the flexibility of higher-order fitting guarantees a wider-range validity of the local hazard approximation. Thus, it enables the inversion of the formulas for practically estimating the allowable demand or the required capacity to fulfill any design objective.

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KEY WORDS: seismic performance evaluation; SAC/FEMA; uncertainty; demand; capacity; probabilistic methods

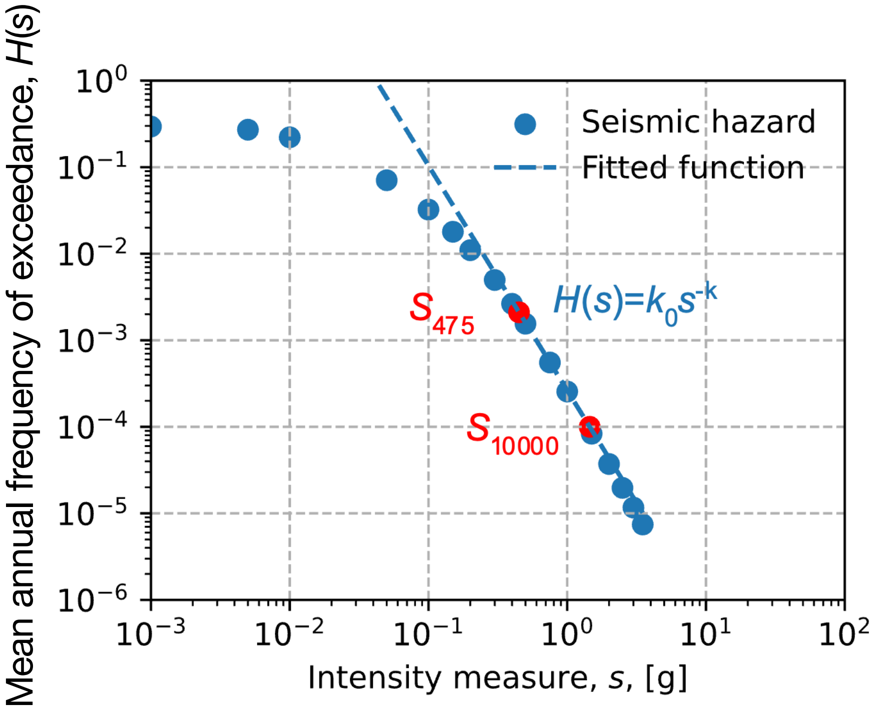
Hazard Fitting Methods

- Constraining the curve at two IM
  - Design basis earthquake – 10% probability of exceedance in 50 years
  - Maximum considered earthquake – 2% probability of exceedance in 50 years

$$k = \frac{\ln\left(H_{DBE}/H_{MCE}\right)}{\ln\left(s_{MCE}/s_{DBE}\right)}$$

$$k_0 = H_{DBE}\left(s_{DBE}\right)^{k}$$

- Accuracy achieved within the constrained IM range – not ideal…
- Large overestimations of hazard at lower and more frequent intensities, in other words where large curvatures are present



Jalayer, F. 2003. “Direct Probabilistic Seismic Analysis: Implementing Non-Linear Dynamic Assessments.” Stanford University.

Hazard Fitting Methods

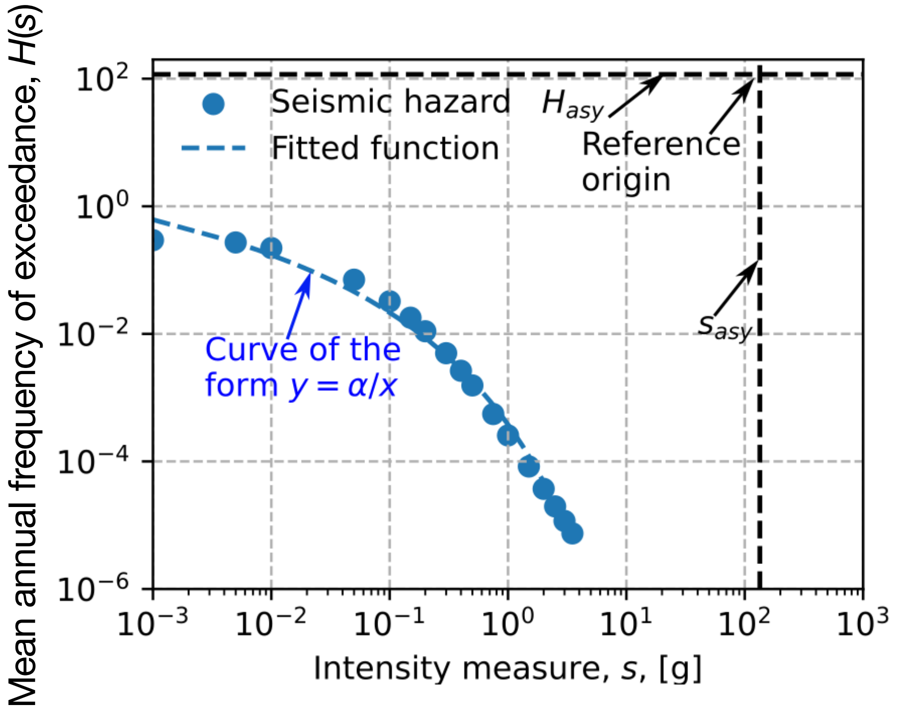
- Improved hyperbolic model using non-linear least-squares regression

$$H(s) = H_{asy} \exp\left[\alpha\left(\ln\left(\frac{s}{s_{asy}}\right)\right)^{-1}\right]$$

- Minimization of relative error between the logarithms of hazard data and fitted curve

$$Minimize\ R = \sum_{i=1}^{n}\left[\ln(H_i) - \ln(H(s_i))\right]^2$$

- Different coefficients to k_0, k_1, k_2 to be determined
- However, not SAC/FEMA compatible, hence not widely utilized within the earthquake engineering community



Bradley, B. A., R. P. Dhakal, M. Cubrinovski, J. B. Mander, and G. A. MacRae. 2007. "Improved seismic hazard model with application to probabilistic seismic demand analysis." Earthq. Eng. Struct. Dyn., 36 (14): 2211–2225. https://doi.org/10.1002/eqe.727.

Proposed Hazard Fitting Methods

- Proposed closed-form expressions to optimize fitting
- Retains the second-order law formulation and the SAC/FEMA compatibility for practical convenience
- No regression or optimization functions are used
- Through the selection of three return periods (default: 5, 20, and 650 years) -> mean annual frequency of exceedance

Inverse

$$\left(r_o, r_1, r_2\right) = \begin{bmatrix} 1 & -\ln\left(s_1\right) & -\ln\left(s_1\right)^2 \\ 1 & -\ln\left(s_2\right) & -\ln\left(s_2\right)^2 \\ 1 & -\ln\left(s_3\right) & -\ln\left(s_3\right)^2 \end{bmatrix}^{-1} \bullet \ln\left(H\right)$$

Matrix Product

$$H_i = \frac{1}{T_{R,i}}$$

$$H = [H_1, H_2, H_3]$$

$$k_0 = \exp\left(r_0\right)$$
$$k_1 = r_1$$
$$k_2 = r_2$$

$$H\left(s\right) = k_o \exp\left(-k_2 \ln^2 s - k_1 \ln s\right)$$

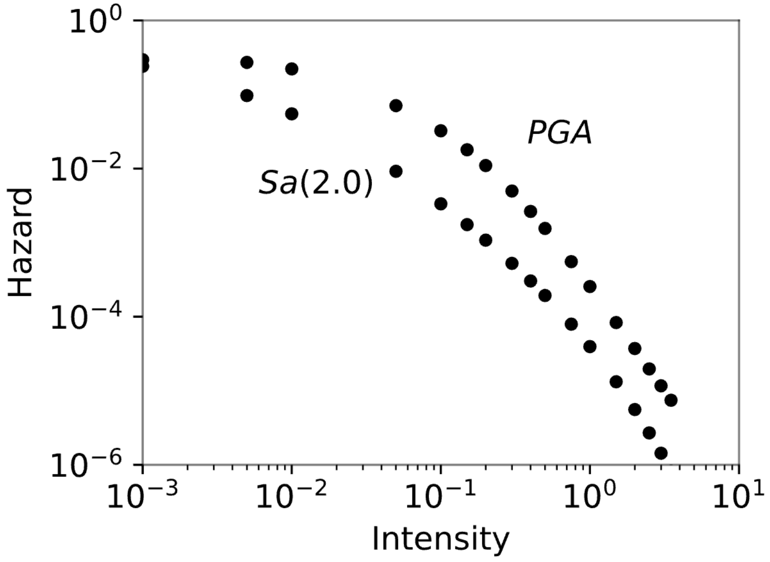
Evaluation of Fitting Methods

- Three distinct locations
  - Italy – PSHA for L'Aquila using OpenQuake with the SHARE hazard model
  - New Zealand – Wellington using the New Zealand seismic hazard model
  - USA – site in California using USGS hazard tool
- Fitting approaches utilized
  - Approach 1 – proposed approach
  - Approach 2a – least-squares fitting of 2nd law and minimizing the following $$Minimize\ R = \sum_{i=1}^{n}\left[\ln\left(H_i\right) - \ln\left(H\left(s_i\right)\right)\right]^2$$
  - Approach 2b – least-squares fitting of 2nd law and minimizing the following $$Minimize\ R = \sum_{i=1}^{n}\left[H_i - H\left(s_i\right)\right]^2$$
  - Approach 3 – log-linear fitting of power law
  - Approach 4 – least-squares fitting of Bradley et al. (2007) approach $$Minimize\ R = \sum_{i=1}^{n}\left[\ln\left(H_i\right) - \ln\left(H\left(s_i\right)\right)\right]^2$$

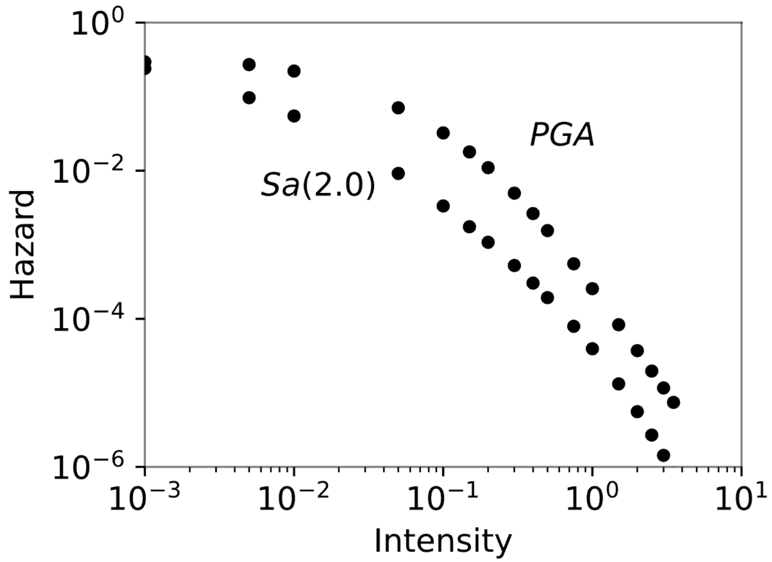
Evaluation of Fitting Methods

- Approach 1 – proposed approach
- Approach 2a – minimization of logarithms of error
- Approach 2b – minimization of error (USA only)
- Approach 3 – power law
- Approach 4 – least-squares of Bradley et al. (2007)

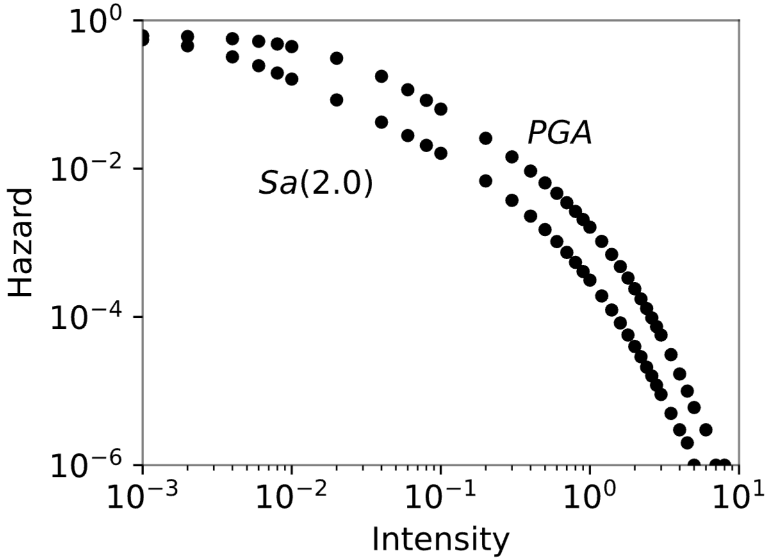
L'Aquila, Italy



California, USA



Wellington, New Zealand



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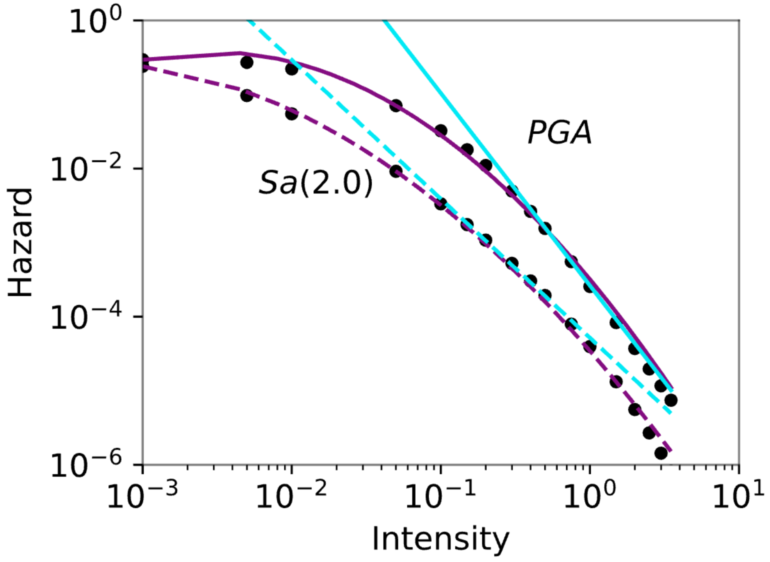
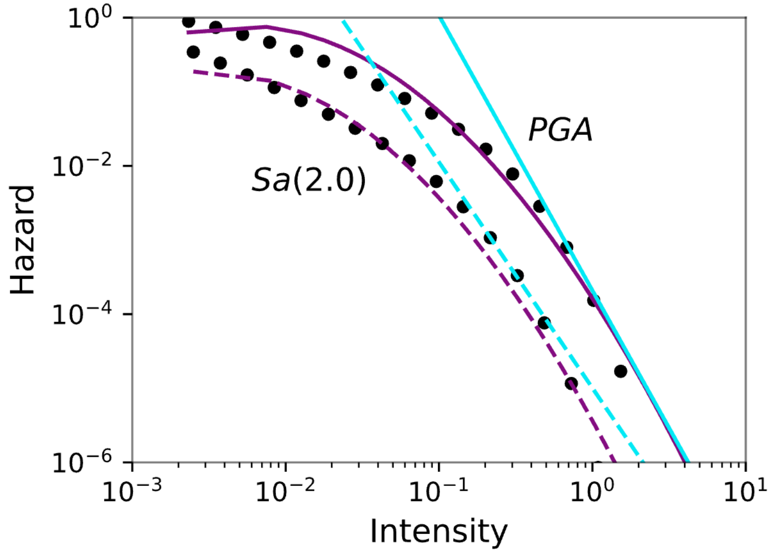
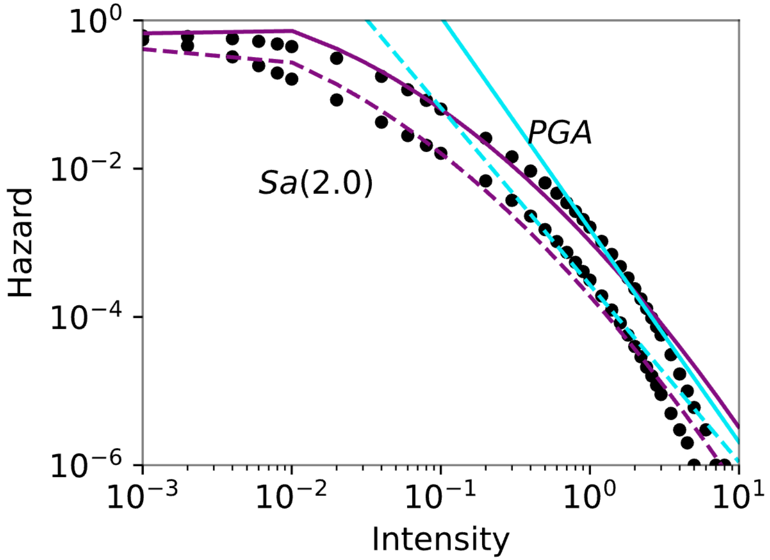


L’Aquila, Italy

California, USA





Wellington, New Zealand

Approach 1

Approach 3

Evaluation of Fitting Methods

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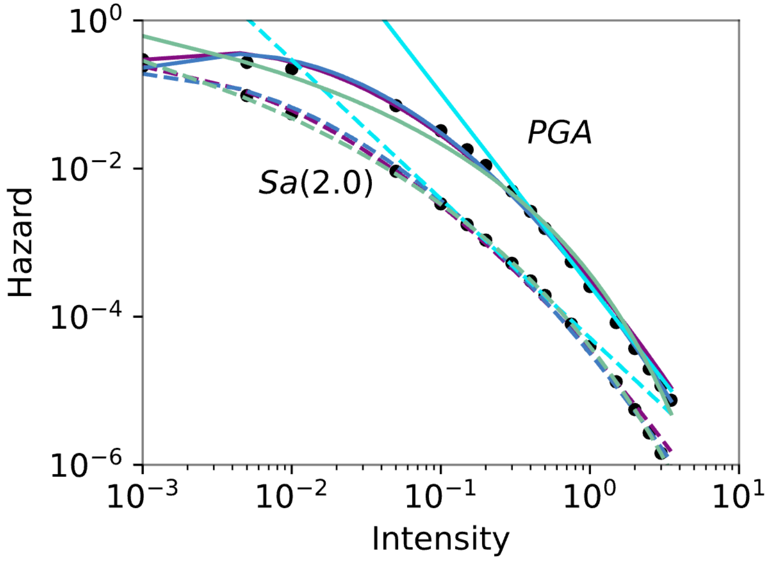
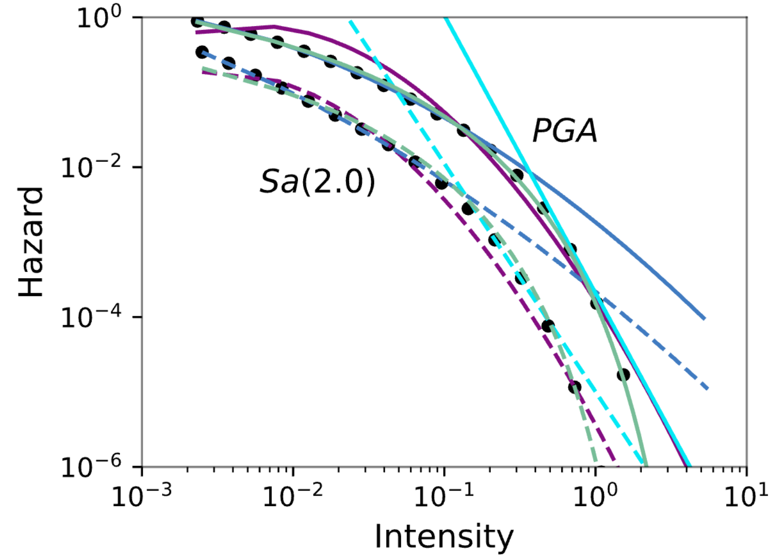
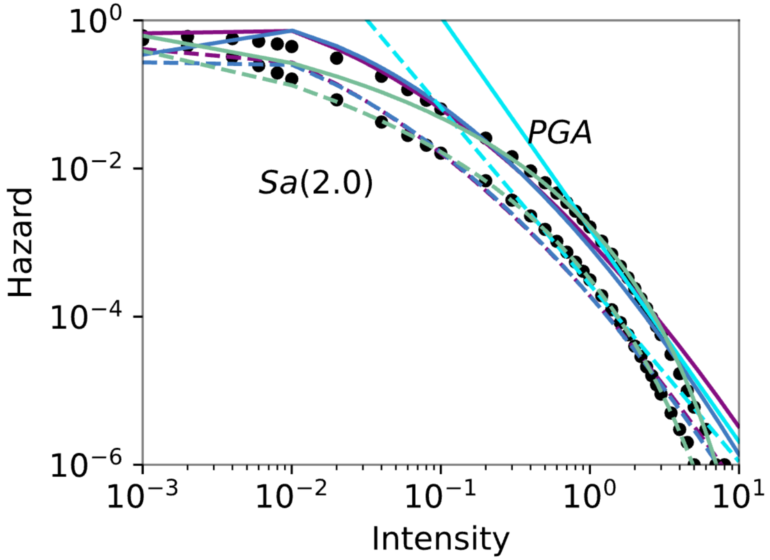


L'Aquila, Italy

California, USA





Wellington, New Zealand

Evaluation of Fitting Methods

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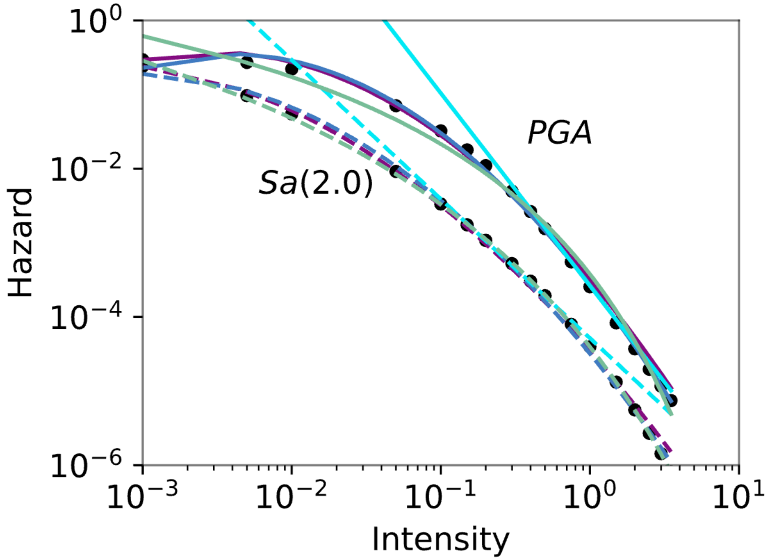
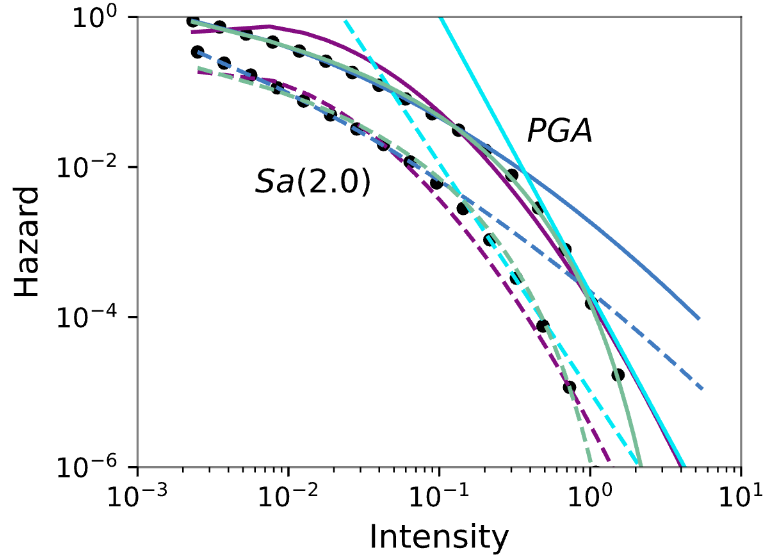
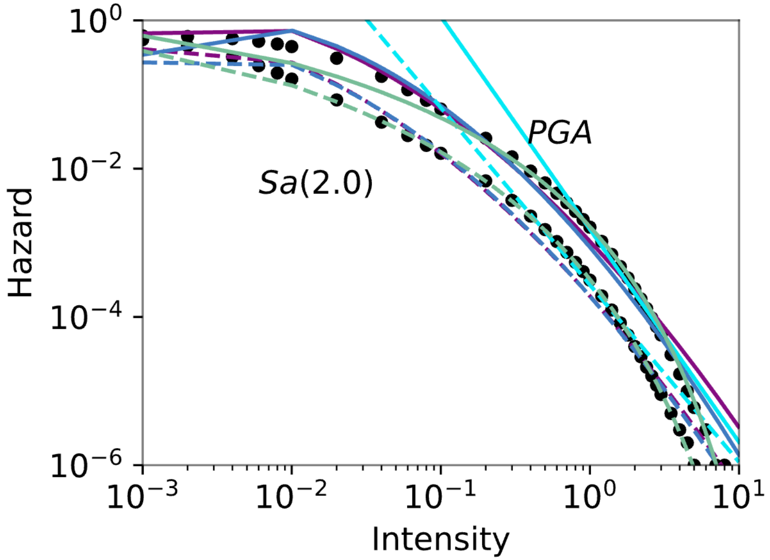


L'Aquila, Italy

California, USA





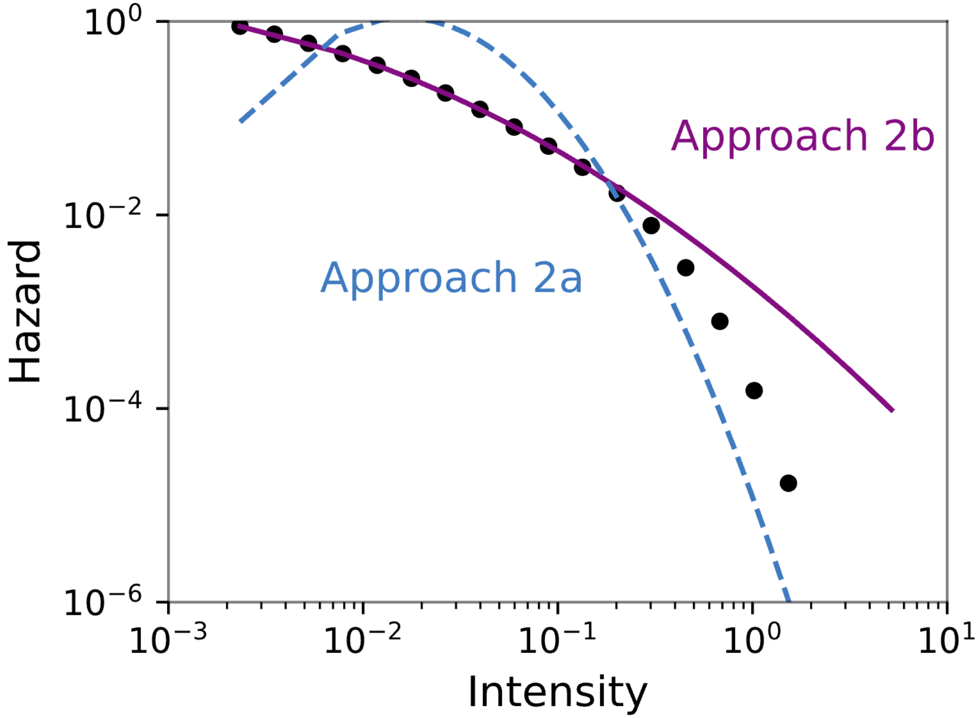
Wellington, New Zealand

Approach 1
Approach 2
Approach 3
Approach 4

Impact of Error Minimization Function

- Significant errors when using the least-squares method on the hazard curve of **California**
- Instead of minimizing the logarithms of the error, minimize the error itself

$$Minimize\ R = \sum_{i=1}^{n}\left[H_i - H(s_i)\right]^2$$



- Approach 2a – minimization of logarithms of error

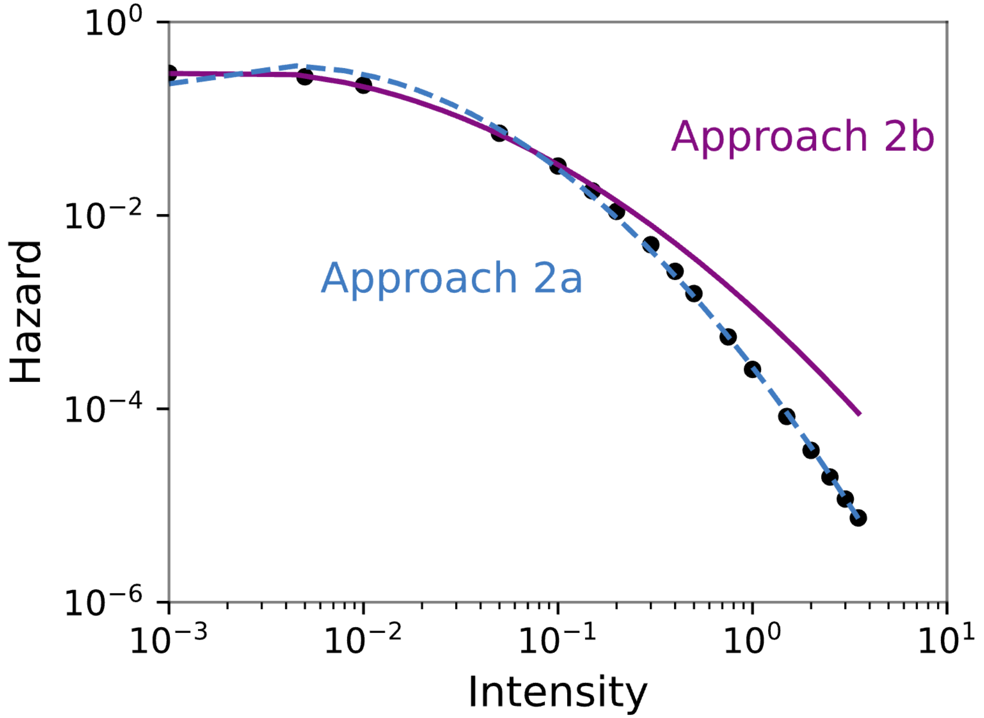
$$Minimize\ R = \sum_{i=1}^{n}\left[\ln(H_i) - \ln(H(s_i))\right]^2$$

- Approach 2b – minimization of error

$$Minimize\ R = \sum_{i=1}^{n}\left[H_i - H(s_i)\right]^2$$

Impact of Error Minimization Function

- Similarly for L'Aquila, **Italy**, it is preferred to use Approach 2a
- One advantage of the proposed approach is the elimination of the need for a minimization function during fitting



- Approach 2a – minimization of logarithms of error

$$Minimize\ R = \sum_{i=1}^{n} \left[\ln\left(H_i\right) - \ln\left(H\left(s_i\right)\right) \right]^2$$

- Approach 2b – minimization of error

$$Minimize\ R = \sum_{i=1}^{n} \left[H_i - H\left(s_i\right) \right]^2$$

Implications on Risk Assessment

- Despite the errors in hazard model predictions, quality of the fits can be further gauged via computation of MAF of exceedance at given peak storey drifts (PSD)
- Demand-intensity models selected to characterize a ductile structure that has a first mode-based beam-sway mechanism
- Closed-form expression to calculate MAF of limit-state exceedance

$$\lambda = H\left[\left(\frac{\theta_c}{m}\right)^{\frac{1}{b}}\right]\exp\left(\frac{k^2}{2b^2}\beta_{Tot}^2\right)$$

- Refinement for 2nd order fitting approach

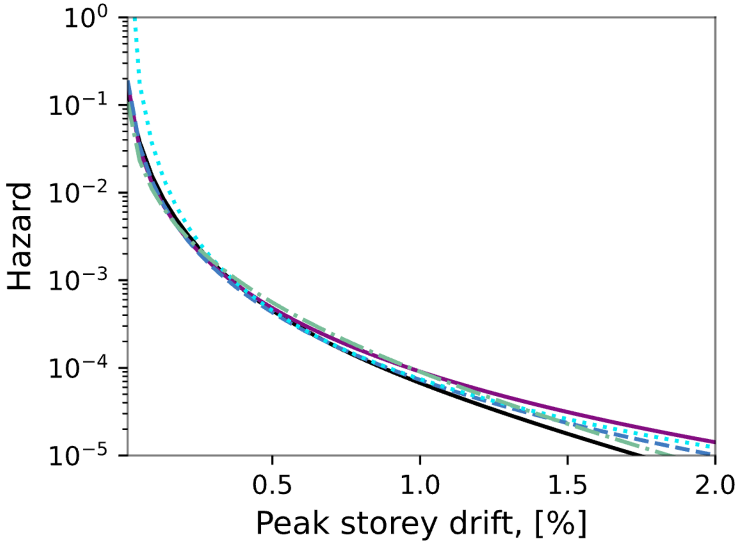
$$\lambda = \sqrt{\phi^{/}}k_0^{1-\phi^{/}}H\left[\left(\frac{\theta_c}{m}\right)^{\frac{1}{b}}\right]^{\phi^{/}}\exp\left(\frac{k_1^2\phi^{/}}{2b^2}\beta_{Tot}^2\right)$$

$$\phi^{/} = \frac{1}{1+2k_2\beta_{Tot}^2/b^2}$$

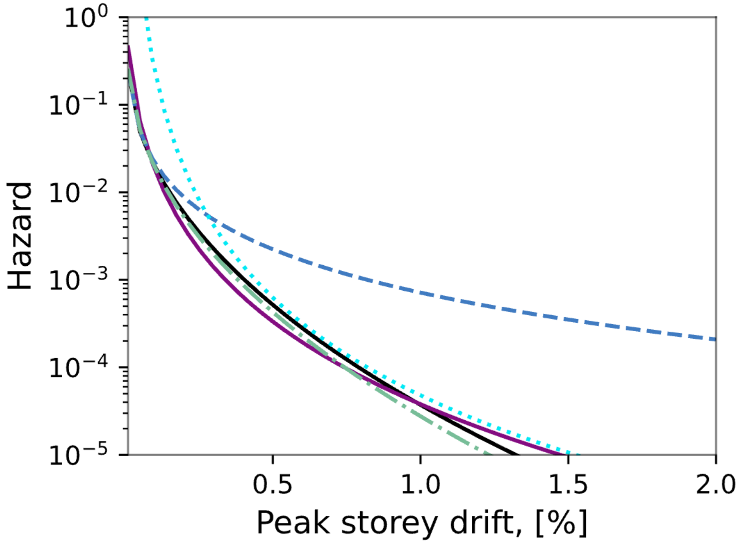
Implications on Risk Assessment

- Bradley et al. (2007) not SAC/FEMA compatible, hence the predicted model was convolved with structural response through direct integration via a trapezoidal rule
- No significant errors in terms of risk computation
- Discrepancies at high and low PSD noted for Approaches 2 and 3
- High errors for California at high PSD values
- The proposed approach may be tuned to target rare events when performing collapse risk assessment

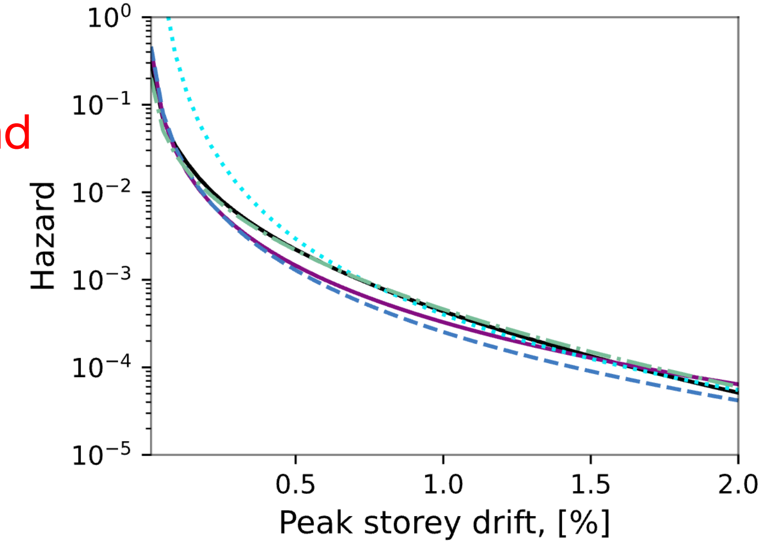
L'Aquila, Italy



California, USA



Wellington, New Zealand



Open-Source Tool

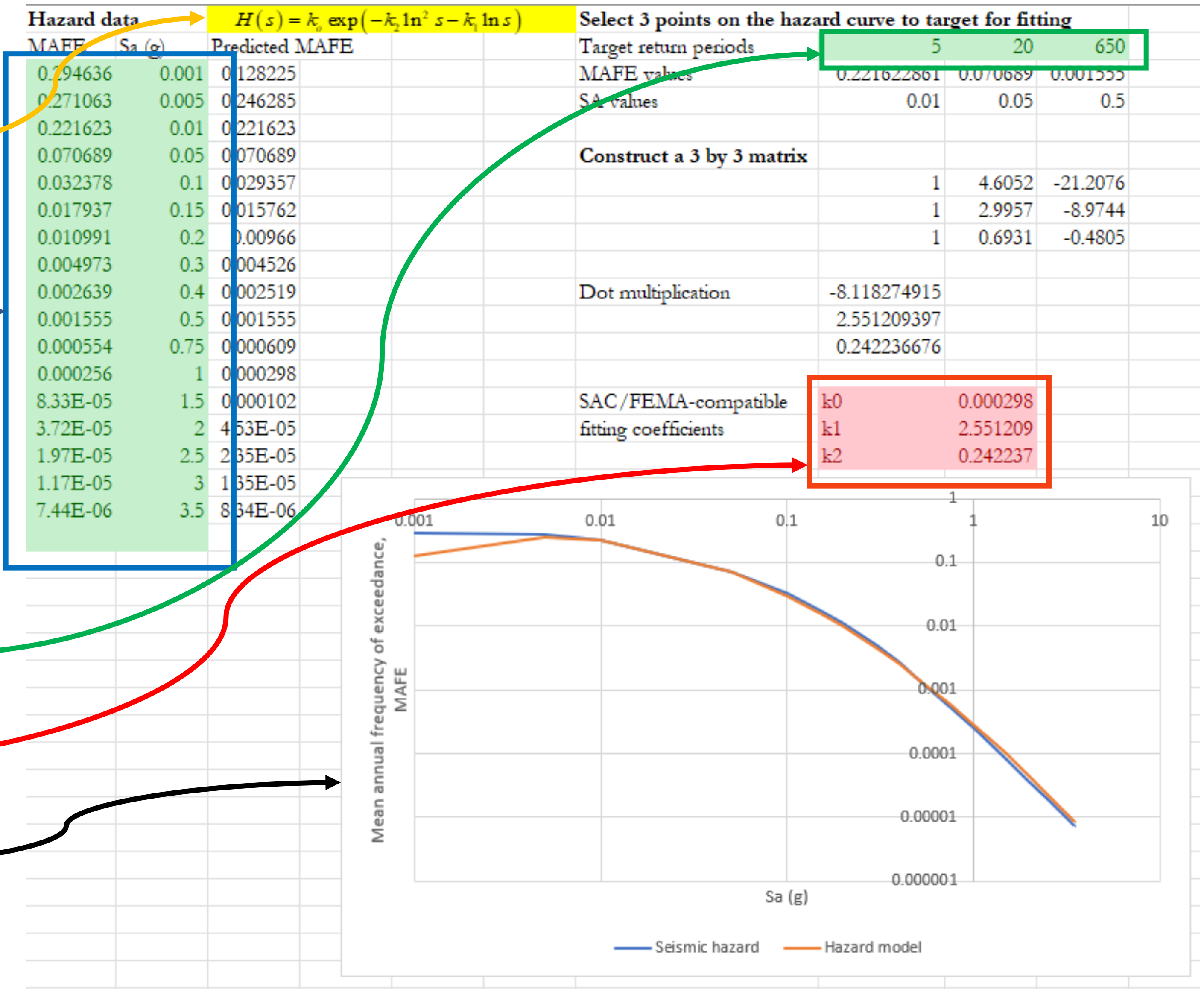
SAC/FEMA-compatible function to fit

Input MAFE and Intensity

Input three return periods

Fitting coefficients

Visualization



Conclusions

- Open-source tool at https://github.com/davitshahnazaryan3/HAZARD/tree/master/fitting_tool
- New closed-form solution to capture seismic hazard independent on the region with minimal error while maintaining SAC/FEAM compatibility
- Validated through an application on three sites around the globe representing different tectonic regions
- Proposed formulations allows targeting three distinct points on the hazard curve to prioritize fitting at different intensity levels
- Allows increased accuracy at different sections of seismic hazard