Fitting improved hazard models for SAC/FEMA-compatible seismic analysis

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ABSTRACT: An improved method of fitting second-order analytical functional forms of seismic hazard compatible with SAC/FEMA-style analysis is presented for use in performance-based earthquake engineering. A novel set of expressions based on fitting at three distinct return periods is proposed and a series of computational tools to automate this are provided. Analysts can simply provide the empirical output of seismic hazard analysis to obtain the best analytical fit. The method is based on a selection of three return periods that can be adjusted depending on the needs with the purpose of error reduction. The proposed model is compared to available fitting functions, such as the least-squares method or based on two return periods that have be typically used to date. Comparative analyses in terms of hazard fit and also the implications on risk estimates indicate error reductions across the entire hazard despite using three return periods for fitting the functional form. The proposed fitting method is envisaged to be used in line with closed-form expressions of SAC/FEMA methodology for a more practice-oriented implementation of seismic risk analysis in design and assessment.

1. INTRODUCTION

One of the cornerstones of earthquake engineering is the accurate quantification of seismic hazard. Seismic hazard describes the exceedance of a given ground motion intensity measure (IM) over a specified period of time; in other words, the relationship between IM level and its annual rate of exceedance, and this is typically obtained by performing probabilistic seismic hazard analysis (PSHA) (Esteva 1967). To simply represent the outputs of PSHA for use in practical settings, Sewell et al. (1996) proposed a power law expression for the relationship:

$$H(s) = k_o s^{-k} \tag{1}$$

where H is the annual rate of exceedance of a ground motion intensity, s; k_0 and k are the constants with the former being the annual rate of exceedance of s = 1g, and the latter being the slope of the hazard function in the log-log domain.

A probabilistic framework has been developed, termed performance-based earthquake engineering (PBEE), at the Pacific Earthquake Engineering Research (PEER) center to capture the performance of structures from elastic response right up to global instability, under ground motion excitations (Cornell and Krawinkler 2000). Several methodologies have been developed over the years, one of which was the SAC project by the Federal Emergency Management Agency (FEMA) with the concept of seismic performance evaluation of structures in a probabilistic manner (Cornell et al. 2002). A closed-form solution to estimate the mean annual frequency (MAF) of limit-state exceedance was devised, convolving seismic hazard with structural response fragility and capitalized on the power law representation of seismic hazard in Eq (1) and the assumption of a lognormally-distributed limit state fragility function. Through this, the aleatory variability caused by natural randomness is directly included, while epistemic uncertainty because of incomplete knowledge can be accounted for through a user-selected level of confidence. Due to its simplicity, it has become the core feature of PBEE in several different contexts and applications, from risk and loss assessment of existing buildings (O'Reilly et al. 2020; Pinto and Franchin 2014), to the risk-targeted design of new structures (Shahnazaryan et al. 2022; Shahnazaryan and O'Reilly 2021; Vamvatsikos et al. 2016).

While this SAC/FEMA developed by Cornell et al. (2002) is simple in its nature, several issues have been outlined, primarily by Aslani and Miranda (2005) and Bradley and Dhakal (2008). The main criticism relates to the adequacy of the power law representation of the hazard curve, as due to the hyperbolic shape of actual hazard analysis outputs, large errors may be introduced through the use of Eq. (1). Bradley et al. (2007) proposed an alternative functions that recognized the asymptotical nature of hazard analysis data in terms of exceedance rates at very low intensities, in addition to the maximum intensity at very long return periods. In addition, Vamvatsikos (2013) revised Eq. (1) to incorporate the curvature of hazard data to give a second-order power law:

$$H(s) = k_o \exp\left(-k_2 \ln^2 s - k_1 \ln s\right) \tag{2}$$

where k_1, k_2>0 and k_2≥0 are the constants, with the latter characterizing the (local) hazard curvature. This closed-form solution provides an improved estimate of demand hazard within the range of exceedance rate where the constant of the equations is fitted.

While Eq. (1) and (2) offer closed-form analytical functions to quantify seismic hazard, and subsequent tools to simply estimate risk, analysts still need some robust and objective way to quantify the coefficients. There are several methods of fitting these coefficients and the accuracy of those methods varies depending on whether a linear or second-order power law is selected, in addition to which fitting method is employed. Additionally, analytical functions exist that capture the seismic hazard adequately but do not maintain the SAC/FEMA compatibility to further their integration into simplified seismic design and assessment. This paper provides a new objective and robust fitting solution to represent the seismic hazard with minimal error, while also maintaining practical implementation for risk analysis.

2. HAZARD FITTING METHODS

2.1. Existing methods

There are several approaches to computing the fitting parameters of Eq. (1) and (2). Jalayer (2003) proposed fitting Eq. (1) by constraining the function at two IM levels: design basis earthquake (DBE) and maximum considered earthquake (MCE) at 10 and 2% probabilities of exceedance in 50 years, respectively. By doing so, the parameters could be computed as follows:

$$k = \frac{\ln\left(H_{DBE}/H_{MCE}\right)}{\ln\left(s_{MCE}/s_{DBE}\right)} \tag{3}$$

$$k_0 = H_{DBE}\left(s_{DBE}\right)^k \tag{4}$$

where s_{DBE}, s_{MCE} are the ground motion intensities, and H_{DBE} and H_{MCE} are the annual rates of exceedance at the DBE and MCE hazard levels. Essentially, the accuracy of the model is only achieved within the constrained IM range (Figure 1), and large overestimations of hazard may result at lower and more frequent intensities if another local fit of Eq. (1) is not conducted.

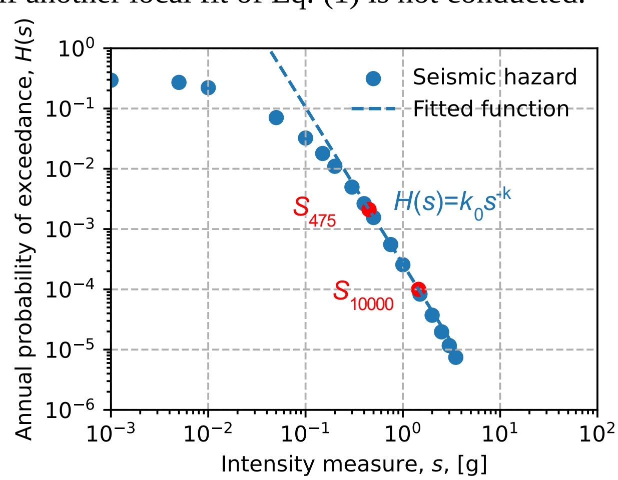

Figure 1: Hazard data fitted through Eq. (1) by constraining at two IM levels.

Bradley et al. (2007) reviewed the use of a simple power law to represent the hazard curve and proposed fitting hazard data to an improved hyperbolic model using non-linear least-squares regression. The primary reason was due to inaccuracies of the power law in capturing the hazard data where large curvatures were present, which tended to introduce large errors in the fit. Given the shape of the hazard data, the curve was approximated through a hyperbola of form $y=\alpha/x$ with vertical and horizontal asymptotes (Figure 2):

$$H(s) = H_{asy} \exp\left[\alpha \left(\ln\left(\frac{s}{s_{asy}} \right) \right)^{-1} \right] \tag{5}$$

where H_{asy}, s_{asy} and α are determined through the fitting. This is done by minimizing the relative error between the logarithms of the hazard data and the fitted curve via:

$$Minimize\ R = \sum_{i=1}^{n} \left[\ln(H_i) - \ln(H(s_i)) \right]^2 \tag{6}$$

where H_i are the hazard data points, and $H(s_i)$ are the values of H obtained from Eq. (5). Even though accurate results can be obtained following this approach, it does not lend itself towards simple estimates of seismic risk like the SAC/FEMA method described by Cornell et al. (2002), and therefore has not been widely utilized within the earthquake engineering community.

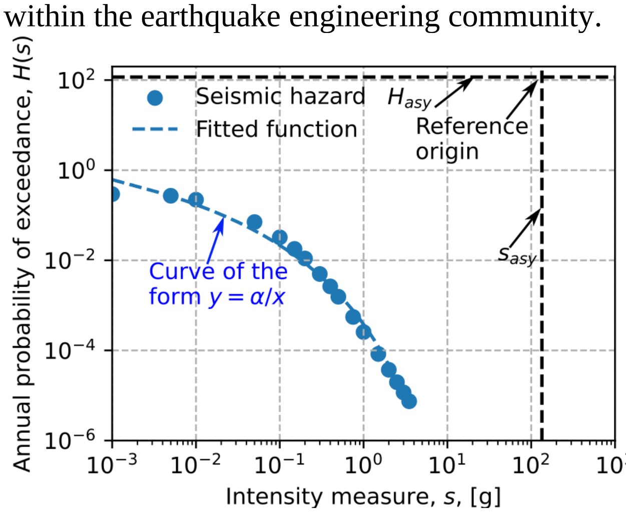

Figure 2: Hazard data fitted following the Bradley et al. (2007) method.

Similarly, Vamvatsikos (2013) aimed to rectify the issues of the power law by incorporating the effects of curvature in the formulation. A higher-fidelity second-order power-law hazard fit was used (Eq. (2)) and new analytical expressions were derived to offer better predictive ability regardless of the shape of the hazard function. The advantage of these new expressions is their SAC/FEMA compatibility as well as the improved accuracy over a simpler power law. However, where very large curvatures of seismic hazard are present, the approach might not perfectly fit the curve, especially at the highest and lowest MAFs. This is not a criticism of the model but rather the fitting approach generally utilized to obtain the coefficients, which is usually a non-linear least-squares regression model available in most analysis tools.

2.2. Proposed method

To remedy the drawbacks of the fitting methods currently available, novel closed-form expressions are proposed to optimize the fitting obtained, while retaining the second-order law formulation and the SAC/FEMA compatibility that offers such practical convenience. The key difference lies with the fitting approach, whereby instead of using optimization functions or regression on the seismic hazard data directly, several analytical expressions are used. Initially, three points are selected on the hazard curve (Eq. (7)), where the fitting function will be optimized (Eq. (2)). The definition of these points is a customizable user input, but default values are given in later sections. The actual hazard data points of IM are then used to construct a 3 by 3 matrix in Eq. (8). Then the matrix product of the inverted matrix and the vector of logarithms of H is obtained via Eq. (9). Finally, the fitting coefficients are obtained via Eq. (10).

$$\begin{aligned} s &= [s_1, s_2, s_3] \\ H &= [H_1, H_2, H_3] \end{aligned} \tag{7}$$

$$r = \begin{bmatrix} 1 & -\ln(s_1)^1 & -\ln(s_1)^2 \\ 1 & -\ln(s_2)^1 & -\ln(s_2)^2 \\ 1 & -\ln(s_3)^1 & -\ln(s_3)^2 \end{bmatrix} \quad (8)$$

$$(r_o, r_1, r_2) = \begin{bmatrix} 1 & -\ln(s_1)^1 & -\ln(s_1)^2 \\ 1 & -\ln(s_2)^1 & -\ln(s_2)^2 \\ 1 & -\ln(s_3)^1 & -\ln(s_3)^2 \end{bmatrix}^{-1} \bullet \ln(H) \quad (9)$$

$$k_0 = \exp(r_0) \quad (10)$$
$$k_1 = r_1$$
$$k_2 = r_2$$

The fitting coefficient may then be inserted into the functional form of Eq. (2). If the fitted data does not match well with the actual seismic hazard, or at certain points, desirable fitting is not achieved, the target points on the hazard curve in Eq. (7) could be updated.

3. EVALUATION OF FITTING METHODS

In order to carry out a comparative evaluation of these different functional forms and fitting approaches, example locations in three different regions were selected: Italy, New Zealand, USA. The goal is to understand how the fitting functions would perform with drastic variations in the curvature of the seismic hazard due to various tectonic regions.

PSHA was performed for L'Aquila in Italy using OpenQuake (Pagani et al. 2014) with the SHARE hazard model (Woessner and Wiemer 2005). The hazard curve of Wellington was obtained through the New Zealand seismic hazard model (Stirling et al. 2012). Finally, the hazard curve of a site located in California was obtained through the USGS hazard tool (2023).

3.1. Hazard fitting

Following PSHA, three hazard curves were selected for comparative purposes and were fitted following four different approaches:

- Approach 1: Proposed fitting approach of Eq. (2) utilizing Eqs. (7) - (10) and no minimization function;
- Approach 2a: Least-squares fitting approach of Eq. (2) and minimizing via Eq. (6);
- Approach 2b: Least-squares fitting approach of Eq. (2) and minimizing via Eq. (11);
- Approach 3: Log-linear fitting approach of Eq. (1) using Eqs. (3) and (4);
- Approach 4: Least-squares fitting approach of Eq. (5) and minimizing via Eq. (6), as advocated by Bradley et al. (2007).

For the site of L'Aquila, all approaches achieved very good fits (Figure 3a), except the log-linear approach due to the slight curvature in the hazard data, which is an expected result. The Wellington fits deviated slightly at low intensities, where a curvature changes significantly after 0.01g (Figure 3b). In contrast, for the site in California, while the error between the observed and predicted hazard is small at high return periods, the error increases dramatically for all approaches, except for Bradley et al. (2007), at smaller return periods (Figure 3c).

3.1.1. Impact of error minimization function

Given the curvature of hazard curves of California, the least-squares method resulted in significant errors, hence instead of minimizing the logarithms of the error, the following error was minimized:

$$\textit{Minimize } R = \sum_{i=1}^{n} \left[H_i - H(s_i) \right]^2 \quad (11)$$

This was done as significant errors were observed when using the least squares approach with Eq. (6). This is predominantly due to the smaller penalization effect at high periods using the natural logarithm. Therefore, when using the least-squares approach, one must carefully select the minimization function, as shown in Figure 4. The significant variation of the predicted hazard curves demonstrates the vital importance of selecting a correct error function when employing the least-squares fitting method. The major advantage of the proposed approach eliminates the need for a minimization function during fitting, providing a key benefit.

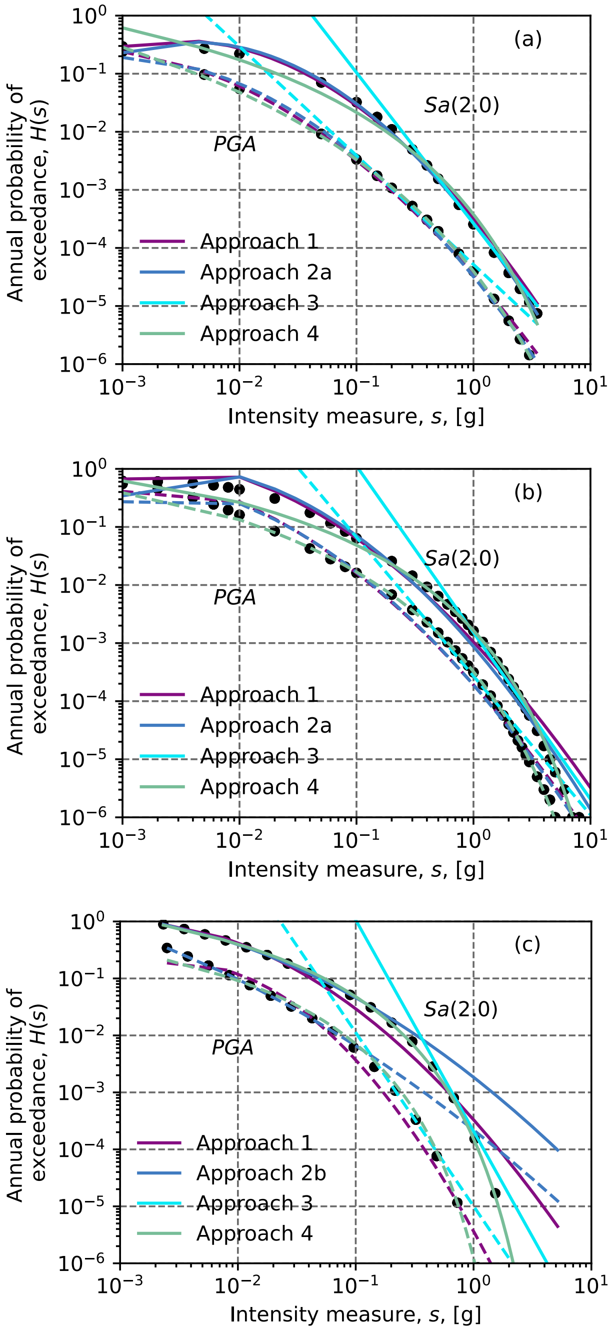

Figure 3: Comparison of fitting functions for the site of (a) L'Aquila, Italy, (b) Wellington, New Zealand, and (c) California, USA, where the fits for PGA and Sa(2s) are shown via the dashed and solid lines, respectively

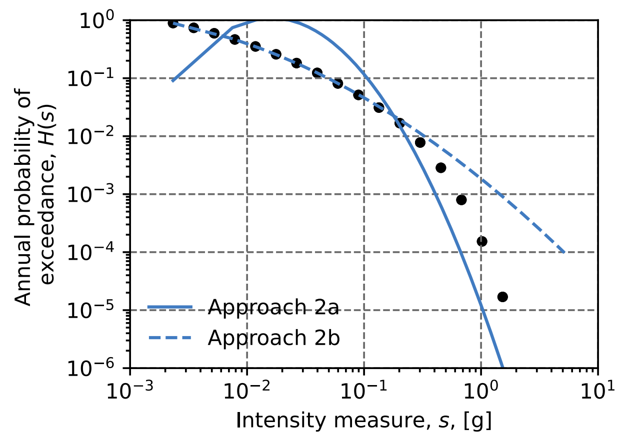

Figure 4: Comparison of least-squares fitting functions for the site of California, USA using different error functions.

3.1.2. Impact of targeted hazard curve range

However, even with a proper selection of an error function, significant variations of observed versus predicted hazard curves are observed at high-intensity levels. To counteract, the proposed approach may be utilized to target better fitting at higher intensity levels. Figure 5 demonstrates the flexibility of the proposed approach to improve the fitting quality at intensities of interest with user-defined target intensity levels as per Eq. (7).

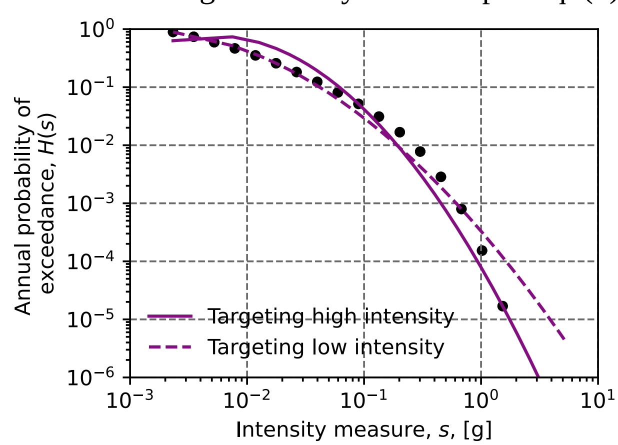

Figure 5: Comparison of the proposed fitting functions (Approach 1) for the site of California, USA using different target intensities: targeting high intensities (5, 20, 59000 years); targeting low intensities (5, 10, 1250 years).

3.2. Implications on risk assessment

Even though the errors in predicting the hazard model via various fitting approaches might produce seemingly large errors as observed in Figure 3, to further gauge the quality of the fits, they are used to compute the MAF of exceedance, λ, at a given peak story drift (PSD), θ, in a building. For that purpose, an exemplary demand-intensity model (O'Reilly and Calvi 2020) was assumed to characterize the performance of a ductile structure that is expected to be a first mode-based beam-sway mechanism.

As previously mentioned, Cornell et al. (2002) devised a closed-form solution to estimate the MAF of limit-state exceedance that convolves the power-law fitting of seismic hazard (Eq. (1)) with the structural response as:

$$\lambda = H\left[\left(\frac{\theta_c}{m}\right)^{\frac{1}{b}}\right]\exp\left(\frac{k^2}{2b^2}\beta_{Tot}^2\right) \tag{12}$$

where θ_C is the PSD where λ is being computed, m and b are the fitting parameters of the demand-intensity model (taken here as m=0.5 and b=1.0), and β_{Tot} is the total uncertainty in the PSD (taken as β_{Tot}=0.4). Vamvatsikos (2013) expanded on this with a refined seismic hazard curve characterized through a second-order fitting of Eq. (2) and proposed:

$$\lambda = \sqrt{\phi'}\,k_0^{1-\phi'}\,H\left[\left(\frac{\theta_c}{m}\right)^{\frac{1}{b}}\right]^{\phi'}\exp\left(\frac{k_1^2\phi'}{2b^2}\beta_{Tot}^2\right) \tag{13}$$

where ϕ' is given by:

$$\phi' = \frac{1}{1+2k_2\beta_{Tot}^2/b^2} \tag{14}$$

The PSD exceedance rates were computed following each of the approaches for all the sites and compared to the direct integration method, where the seismic hazard output from PSHA was directly convolved with the assumed demand-intensity model, giving a reference value with which to evaluate the different approaches. Since the functional form of Bradley et al. (2007) of Eq. (5) is not SAC/FEMA-compatible, the predicted model was convolved with the structural response through direct integration via the trapezoidal rule.

Figure 6 shows the computed exceedance rates of PSD following the different approaches.

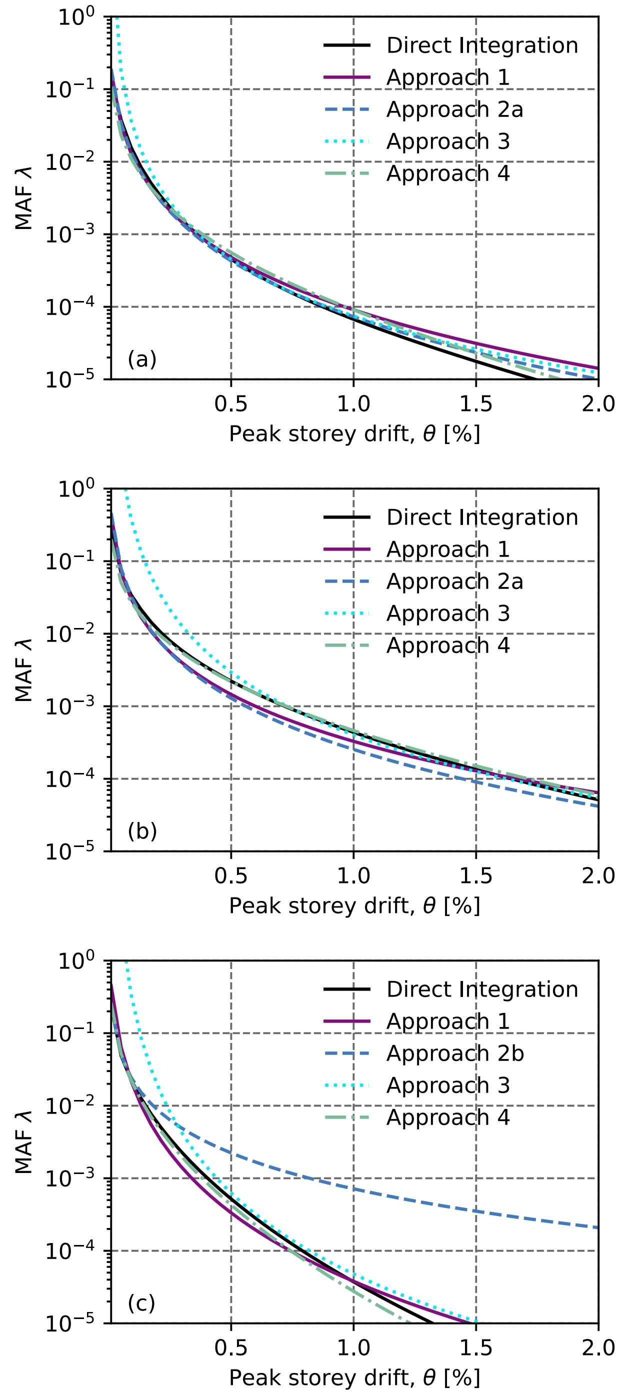

Figure 6: PSD exceedance rates for the site of (a) L'Aquila, Italy, (b) Wellington, New Zealand, and (c) California, USA for PGA.

It can be seen that the errors are not as significant in terms of the risk calculation, when compared to the hazard shown in Figure 3. However, discrepancies at high and low values of PSD are noted for Approaches 2 and 3.

Additionally, the errors begin to significantly increase for the site in California, especially for high PSD values that could characterize collapse. Therefore, to avoid potential issues the proposed approach might be tuned to target rare events, i.e., high IM values when performing collapse risk assessment.

4. AVAILABLE TOOLS

Given the robust and user-friendly nature of the proposed approach for fitting the seismic hazard, several tools are made available to the reader at https://github.com/davitshahnazaryan3/HAZARD/tree/master/fitting_tool. The user is required to input a seismic hazard data points obtained through PSHA. By default, the tool fits to data points at return periods of around 5, 20, and 650 years, but these values are easily adjustable to meet the user's specific needs and improve conformance of results.

5. CONCLUSIONS

Different approaches for fitting seismic hazard curves were discussed within this study in a comparative setting. New closed-form fitting solutions were devised to capture the seismic hazard with minimal error while maintaining SAC/FEMA compatibility. Three sites around the globe characterizing different tectonic regimes resulting in different seismic hazard curve shapes were utilized to gauge the accuracy of different hazard fitting methods. The sites included L'Aquila, Italy; Wellington, New Zealand; and California, USA. Several conclusions were derived from the study:

1. The proposed formulation allows targeting three distinct points on the seismic hazard curve to prioritize fitting at different intensity levels. In contrast to the log-linear power law, the proposal approach bases itself upon three points to mimic a hyperbolic curve.
2. The target intensity measure levels may be tuned to increase accuracy at different sections of seismic hazard. In contrast, the typical least-square fitting approach might result in high errors at collapsing intensities forcing a manual-based fitting approach and preventing automation.
3. The least-squares fitting approach is very sensitive to the error function selection. For example, for the site in the USA, the error was minimized, while for the site in Italy, the logarithms of the error were minimized, whereas vice versa gave poor fits.
4. Finally, the proposed method can be easily adapted through hand calculations rather than using a relatively complex nonlinear fitting approach such as a least-squares method.

6. ACKNOWLEDGEMENTS

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